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Limit cycles in planar piecewise linear differential systems with nonregular separation line

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HIGHLIGHTS

- The nonregularity in the separation line occurs only in one point.
- The number of periodic orbits is bigger than for the regular case.
- All the periodic orbits have the breaking point in its interior.
- Higher Melnikov theory is used for the described bifurcating phenomena.
- The stabilization phenomena in the number appear increasing the order.

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ABSTRACT

In this paper we deal with planar piecewise linear differential systems defined in two zones. We consider the case when the two linear zones are angular sectors of angles α and $2\pi-\alpha$, respectively, for $\alpha \in (0, \pi)$. We study the problem of determining lower bounds for the number of isolated periodic orbits in such systems using Melnikov functions. These limit cycles appear studying higher order piecewise linear perturbations of a linear center. It is proved that the maximum number of limit cycles that can appear up to a sixth order perturbation is five. Moreover, for these values of α, we prove the existence of systems with four limit cycles up to fifth order and, for $\alpha = \pi/2$, we provide an explicit example with five up to sixth order. In general, the nonregular separation line increases the number of periodic orbits in comparison with the case where the two zones are separated by a straight line.

1. Introduction

Many systems of relevance to applications are modeled using piecewise linear differential systems. The study of such systems goes back to Andronov and coworkers [1] and nowadays still continues receiving attention by many researchers. For more details about piecewise linear (and piecewise smooth in general) differential systems see for instance the books of Filippov [2] and di Bernardo et al. [3] and the references quoted therein.

In the classical theory for smooth systems an important topic is the weak 16th Hilbert's problem. The question is: Which is the maximum number of isolated periodic orbits, also called limit cycles, that bifurcate perturbing a center? This problem for piecewise differential systems defined in two zones have been studied recently, among other papers, in [4–12]. Usually the separation line between the two zones is a straight line. Here we study the case when the separation line is nonregular. angular regions, i.e. the separation line is formed by two semi-straight lines that coincide at the origin forming an angle α, with $\alpha \in (0, \pi)$. In particular we provide lower bounds for the number of limit cycles of the linear center under perturbation, with piecewise linear vector fields, up to order six. After a linear transformation, if it is necessary, it is not restrictive to assume that the center is the classic harmonic oscillator. More precisely, for each

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