# PERIODIC ORBITS IN PREDATOR-PREY SYSTEMS WITH HOLLING FUNCTIONAL RESPONSES

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ABSTRACT. We study the existence of periodic orbits of the predator–prey systems $\dot{x} = rx - f(x)y$, $\dot{y} = \big(g(x) - \mu\big)y$, for different types of Holling functional responses $f(x)$ of the predator. For the first type we have centers, for the second type there is neither periodic orbits nor limit cycles, and for the third and fourth types there are limit cycles.

**1 Introduction** Since the seminal work of Kolmogorov [8], extensive work has been done on the study of the dynamic of a predator-prey system modeled by two autonomous differential equations. One very popular version of such a system, the so called Gause type model, has the following general form

$$\begin{aligned} \dot{x} &= xh(x) - f(x)y, \\ \dot{y} &= (cf(x) - d(y))y, \end{aligned} \tag{1}$$

As usual the dot denotes derivative with respect to the time variable $t$.

The global stability of the system is typically determined by the existence of a positive attractor, either an equilibrium or a limit cycle. For this reason, the existence and uniqueness of positive attractors and limit cycles of system (1) has attracted much interest in recent years. For a sample of these studies, see Cheng [3], Xiao and Zhang [15], Hazík [6], Kuang [10], Moghadas [12]. Most of the recent work has employed technical methods, such as transforming the system to an equivalent generalized Lienard system or trying directly to exploit the special structure of the limit cycle and the prey isocline. The models also incorporated non monotonic functional response to simulate defence mechanisms of the prey, see for example González et al [11], Ruan and Xiao [13], Wolkovicz [14], Xiao and Zhang [16], Zhu et al [17].

Roughly speaking all the models which one finds in the literature consider a function $h(x)$ such that $(x - K)h(x) < 0$ for all $x \geq 0$, as in the logistic growth model. The case of exponential growth of the prey has not been considered. Levin in [9] investigated the effect of the density–dependent predator death rate upon the stability of equilibria for the model

$$\begin{aligned} \dot{x} &= ax - f(x)y, \\ \dot{y} &= (cf(x) - d(y))y, \end{aligned} \tag{2}$$

where $d$ is an increasing function and $d(0) > 0$. However the existence of limit cycles was not investigated.

In this work we will study the predator–prey model

$$\begin{aligned} \dot{x} &= rx - f(x)y, \\ \dot{y} &= \big(g(x) - \mu\big)y, \end{aligned} \tag{3}$$

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