Invariant Manifolds Associated to Homothetic Orbits in the n-body Problem

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1. Introduction. Our main goal in this paper is to study the global behavior of the homothetic solutions of the planar and spatial n-body problem which begin and end in a simultaneous collision of all the particles. A similar study for the collinear n-body problem has been made by Devaney [2]. Following Devaney we consider the homothetic solutions as heteroclinic orbits of a dynamical system connecting two hyperbolic fixed points. This system is obtained regularizing the n-body problem using a technique of McGehee [4] where the singularity of total collision is blown up and in its place is glued an invariant total collision manifold. Furthermore, scaling the time, the solutions which previously reached total collision in finite time are made to tend asymptotically toward hyperbolic equilibrium points for the flow on the total collision manifold. We summarize the changes of variable and resulting equations (see [4], [8], [2]).

Let $q = (q_1,\ldots,q_n)^t$, $p = (p_1,\ldots,p_n)^t$ where q_i, $p_i \in R^k$ are the position and momentum of the body with mass m_i. Let

$$M = \operatorname{diag}(\underbrace{m_1,\ldots,m_1}_{k},\ldots,\underbrace{m_n,\ldots,m_n}_{k}),$$

$$B = \operatorname{diag}(\underbrace{I_k,\ldots,I_k}_{n}),$$

$$A = BM,$$

$$T(p) = \frac{1}{2}p^tM^{-1}p \quad \text{(kinetic energy)},$$

$$U(q) = \sum_{1\le i<j\le n} \frac{m_im_j}{|q_i-q_j|} \quad \text{(potential energy)},$$

where I_k is the identity matrix in R^k. The equations of motion are

$$\dot q = M^{-1}p,$$
$$\dot p = \nabla U(q),$$

with the first integrals:

Indiana University Mathematics Journal ©, Vol. 31, No. 4 (1982)