Lefschetz zeta functions and forced set of periods

JOSEFINA CASASAYAS, JAUME LLIBRE, AND ANA NUNES

ABSTRACT. We study C^1 self maps of compact manifolds whose periodic points are transversal and obtain information on the set of periods from the associated Lefschetz zeta function.

1. Introduction and basic definitions

Given a continuous self-map of a compact manifold M of dimension n, its *Lefschetz number* is defined as

$$L(f) = \sum_{k=0}^{n} (-1)^k \mathrm{tr}\ (f_{*k}),$$

where $f_{*k} : H_k(M; \mathbb{Q}) \to H_k(M; \mathbb{Q})$ is the endomorphism induced by f on the k-th rational homology group of M.

One of the most useful results to prove the existence of fixed points or, more generally, periodic points for continuous maps $f : M \to M$ in a given homotopy class is Lefschetz fixed point theorem, which says that if $L(f) \neq 0$ then f has a fixed point.

When studying the periodic points of f, i.e., the set

$$\mathrm{Per}(f) = \{m \in \mathbb{N} : f \text{ has a periodic orbit of least period } m\},$$

it is convenient to consider the *Lefschetz zeta function* of f,

$$Z_f(t) = \exp \left(\sum_{m=1}^{\infty} \frac{L(f^m)}{m} t^m \right),$$