CENTRAL CONFIGURATIONS OF THE PLANAR $1 + N$ BODY PROBLEM

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Abstract. In this paper, we give a new derivation of the equations for the central configurations of the $1 + n$ body problem. In the case of equal masses, we show that for n large enough there exists only one solution. Our lower bound for n improves by several orders of magnitude the one previously found by Hall.

Key words: Central configurations, N-body restricted problem.

1. Introduction

The problem of finding the central configurations of a system of N particles in the plane has been the subject of many papers during the last two hundred years. One of the reasons why central configurations are interesting is that they allow us to construct exact solutions of the N-body problem. This was already pointed out by Laplace and, historically, the problem of central configurations was first formulated in this context.

But central configurations also have other interesting properties which show their relevance for the study of the N-body problem. Smale proved that there exists a relation between central configurations and the bifurcations of the surfaces of constant energy and angular momentum. He showed that, in the planar case, the set of classes of central configurations, modulus rotations and rescaling, completely determines the critical values of the energy and angular momentum map [16]. Smale's paper marks the beginning of a renewed interest in this old problem, and many contributions have appeared in the last two decades, namely concerning Smale's conjecture on the finiteness of the number of classes of central configurations and the question of obtaining bounds for that number in some particular cases [4, 8, 10–13, 15, 17], and concerning the relation between central configurations and limit escape and collision configurations [1, 6, 14]. However, the problem is still far from being solved.

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