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Periodic orbits of transversal maps

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1. *Introduction*

One of the most useful theorems for proving the existence of fixed points, or more generally, periodic points of a continuous self-map f of a compact manifold, is the Lefschetz fixed point theorem. When studying the periodic points of f it is convenient to use the Lefschetz zeta function $Z_f(t)$ of f, which is a generating function for the Lefschetz numbers of all iterates of f. The function $Z_f(t)$ is rational in t and can be computed from the homological invariants of f. See Section 2 for a precise definition. Thus there exists a relation, based on the Lefschetz fixed point theorem, between the periodic points of a self-map of a manifold $f:M \to M$ and the properties of the induced homomorphism f_{*i} on the homology groups of M. This relation has been used in several papers, namely [F1], [F2], [F3] and [M]. In these papers, sufficient conditions are given for the existence of infinitely many periodic points in the case when all the zeros and poles of the associated Lefschetz zeta function are roots of unit. Here we restrict ourselves to maps defined on manifolds with a certain homology type. For transversal maps f defined on this class of manifolds, it is possible to extend the techniques introduced in [F1], [F3] and [M] in order to obtain information on the set of periods of f. We recover the above mentioned results of J. Franks and T. Matsuoka, and derive new results on the set of periods of f when the associated Lefschetz zeta function has zeros or poles outside the unit circle.

We shall study C^1 self-maps f of a compact manifold which have only transversal periodic points, so called because the graph of f^m is transverse to the diagonal for all $m > 0$. Following Franks ([F1], [F3]), for such maps the Lefschetz zeta function can be expressed as an infinite product of terms of the form $(1 \pm t^p)$, each one contributed by a single periodic orbit, and one can separate out the contribution to such a product of all the terms of the form $(1 \pm t^{2^m r})$, for a fixed odd integer $r \geqslant 1$. This decomposition in an infinite product and its algebraic properties will be our main tools to study the periodic orbits of f. For more details see Sections 2 and 3.

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