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Simultaneous Periodic Orbits Bifurcating from Two Zero-Hopf Equilibria in a Tritrophic Food Chain Model

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ABSTRACT

We are interested in the coexistence of three species forming a tritrophic food chain model. Considering a linear grow for the lowest trophic species or prey, and a type III Holling functional response for the middle and highest trophic species (first and second predator respectively). We prove that this model exhibits two small amplitud periodic solutions bifurcating simultaneously each one from one of the two zero-Hopf equilibrium points that the model has adequate values of its parameters. As far as we know, this is the first time that the phenomena appear in the literature related with food chain models.

Keywords: Periodic Orbit; Averaging Theory; Zero-Hopf Bifurcation; Population Dynamics

1. Introduction

In general, the Hopf bifurcation is a useful tool to analyse the existence of limit cycles in predator-prey interaction models. For instance, in [1] the authors proved the existence, uniqueness and nonexistence of limit cycles in a predator-prey model considering a strong Allee effect in a prey. In [2], it is considered that a model of three species competes for three resources and it is proved that the existence of two limit cycles evolves the coexistence equilibrium point, and other example is [3]. In a food web the Hopf bifurcation is also the principal tool for proving the coexistence of species that compose the food chain. In this direction Freedman and Waltman [4] studied the persistence of species in a three-level food chain model. They introduce a relative general model, and criteria for the boundedness and stability are established. They consider a Lotka-Volterra predation with a carrying capacity at the lowest level via a logistic map and with a Holling functional response type II predation at the level of the first predator. They gave sufficient conditions for persistence of all three species. Later on, in [5] Freedman and So established criteria for which a simple food-chain model had a globally stable positive equilibrium and also developed criteria in order that such a food chain model exhibited uniform persistence (see also [6]). In these articles, the possibility of existence of limit cycles is important, however it was not studied.

Recently Françoise and Llibre analyse a model representing a tritrophic food chain composed of a logistic prey, a Holling type II predator and a Holling type II top-predator in [7]. Using the averaging theory (see [8-10]) they prove the existence of a stable periodic orbit contained in the region of coexistence of the three species in a tritrophic chain. For some values of the parameters three limit cycles born via a triple Hopf bifurcation. One is contained in the plane where the top-predator is absent. Another one is not contained in the domain of interest where all variables are positive and the third one is contained where the three species coexist. In the literature, there are many papers dedicated to find these types of limit cycles which came from a Hopf bifurcation, but in all these papers the existence of a triple Hopf bifurcation was not proved analytically, see for instance [11-16].

In this paper we analyse a tritrophic food chain model considering Holling functional response of type III for middle and top trophic level and linear grow for the lowest tropic level.

Accordingly with the previous works a general tritrophic food chain model has the form