THE PSEUDO-HOPF BIFURCATION FOR PLANAR DISCONTINUOUS PIECEWISE LINEAR DIFFERENTIAL SYSTEMS

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ABSTRACT. The creation or destruction of a crossing limit cycle when a sliding segment changes its stability, is known as pseudo-Hopf bifurcation. In this paper, under generic conditions, we find an unfolding for such bifurcation, and we prove the existence and uniqueness of a crossing limit cycle for this family.

1. INTRODUCTION

The study of limit cycles is one of the most important problem in the qualitative theory of ordinary differential equations, however, the proof of their existence are generally very complicated. A large list of papers about the arising of limit cycles in piecewise smooth systems in the plane can be found in the literature of recent years, and in these some techniques has been developed to find them. In smooth systems there is a well known mechanism to search for the occurrence of limit cycles, the Hopf bifurcation theorem, see [13, 19]. There are analogous results for piecewise smooth systems, for the case of continuous systems see for example [6, 7, 26, 27], and for the case of discontinuous systems see [1, 8, 11, 12, 14, 18]. In the discontinuous ones we can have more than one limit cycle, either all crossing cycles or including one sliding cycle, and in fact, the determination of the number of limit cycle has been the subject of several recent papers, see [2, 3, 4, 10, 15, 16, 17, 20, 22, 23, 24].

When the appearance of more than one limit cycle is considered, often the mechanism to obtain one of them is by the collision of *two invisible tangencies*. This is, the creation or destruction of one crossing limit cycle occurs when a sliding segment changes its stability, this phenomenon is presented without demonstration in [18] and called *pseudo-Hopf bifurcation*. The appearance of a crossing limit cycle may occur in cases where there is not sliding segment, see [9, 21, 25].

In this paper we find an unfolding for the *pseudo-Hopf bifurcation* for planar discontinuous piecewise linear (DPWL) systems with two zones separated by a straight line. We prove the existence and uniqueness of a crossing limit cycle for all possible dynamic scenarios. It is important to mention that the unfolding found has seven parameters, but at moment that the dynamics on each zone be established, it reduces to five. However, in our result it will not be necessary to establish a priori the dynamics in each zone.

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