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Bifurcation of the separatrix skeleton in some 1-parameter families of planar vector fields

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Abstract

This article deals with the bifurcation of polycycles and limit cycles within the 1-parameter families of planar vector fields X_m^k, defined by $\dot{x} = y^3 - x^{2k+1}$, $\dot{y} = -x + my^{4k+1}$, where m is a real parameter and $k \geq 1$ is an integer. The bifurcation diagram for the separatrix skeleton of X_m^k in function of m is determined and the one for the global phase portraits of $(X_m^1)_{m\in\mathbb{R}}$ is completed. Furthermore for arbitrary $k \geq 1$ some bifurcation and finiteness problems of periodic orbits are solved. Among others, the number of periodic orbits of X_m^k is found to be uniformly bounded independently of $m \in \mathbb{R}$ and the Hilbert number for $(X_m^k)_{m\in\mathbb{R}}$, that thus is finite, is found to be at least one.

MSC: 34C23; 34C07; 37G10; 37G15

Keywords: Rotated vector field; Global phase portrait; Separatrix skeleton; Limit cycle; Hilbert's 16th problem; Nilpotent center problem

1. Introduction

This article concerns periodic orbits and separatrix cycles for the 1-parameter families $(X_m^k)_{m\in\mathbb{R}}$, where X_m^k are planar polynomial vector fields of degree $4k+1$, given by

$$\dot{x} = y^3 - x^{2k+1},\ \dot{y} = -x + my^{4k+1} \tag{1}$$

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