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Hopf-Takens bifurcations and centres

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Abstract

The paper deals with Hopf-Takens bifurcations, both in generic families and in families containing centres. Attention goes to the relation between normal forms, Lyapunov coefficients and the Bautin ideal. In case the Bautin ideal has only one generator we pay attention to the first non-zero Melnikov function.

Keywords: Hopf singularity; Centers; Limit cycles; Generic bifurcation diagrams; Normal forms; Lyapunov coefficients; Bautin ideal; Melnikov functions

1. Introduction

The Hopf bifurcation, also called Andronov-Hopf bifurcation is a very well-known generic and structurally stable one-parameter bifurcation. It unfolds a non-degenerate singularity of codimension 1 and it gives birth to a limit cycle. Since it can be determined by algebraic techniques (positioning the singularity and calculating the 3-jet of the unfolding) it gives rise to a powerful instrument to detect small amplitude periodic dynamics. As such, its use goes beyond the theory of ordinary differential equations.

A generalisation giving rise to more than one limit cycle and to related multiple limit cycle bifurcations has been studied in [T]. The generic p-parameter structurally stable bifurcation is called *Hopf bifurcation* or *Hopf-Takens bifurcation of*

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