THE USE OF MELNIKOV FUNCTIONS IN MULTI-DIMENSIONAL PARAMETER FAMILIES: ALGEBRAIC CURVES OF MAXIMAL CYCLICITY

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It is well known that the cyclicity in one parameter families of planar vector fields can be obtained by computation of Melnikov functions. In this note, the question is addressed if and how one can transfer results obtained in one parameter subfamilies to multi parameter families of planar vector fields.

1. Introduction

We deal with analytic families of planar vector fields $(X_\lambda)_\lambda$, with $\lambda \in \mathbb{R}^p$. We suppose that X_{λ_0} is a vector field of center type. This means that the phase portrait consists of non-isolated periodic orbits. Let Γ be such a non-isolated periodic orbit of X_{λ_0}. Our attention then goes to *isolated* perodic orbits γ or so-called *limit cycles* of X_λ, where λ is close to λ_0 and γ is close to Γ (for the Haussdorf distance [1]). We are especially interested in the maximum number of limit cycles that can originate from Γ after small perturbations of X_{λ_0}. This local upperbound is called the *cyclicity of* $(X_\lambda)_\lambda$ *along* Γ *at* λ_0, and is defined more precisely as:

$$Cycl\,(X_\lambda, (\Gamma, \lambda_0)) = \limsup_{\gamma \to \Gamma, \lambda \to \lambda_0} \{\text{number of limit cycles } \gamma \text{ of } X_\lambda\}$$

There are several techniques in bounding this number [6,10,8,9,11,12]. Traditionally, an analytic family of *displacement maps* is associated to the family $(X_\lambda)_\lambda$ [11,12]. This family of maps is defined by the difference map of the identity Id and the Poincaré-map (or first return map) P_λ :

$$\delta : I \times W \to \mathbb{R} : (s, \lambda) \mapsto \delta_\lambda(s) = P_\lambda(s) - s$$