Available online at www.sciencedirect.com
ScienceDirect
J. Differential Equations 244 (2008) 1359–1394
Journal of Differential Equations
www.elsevier.com/locate/jde

Hilbert's 16th problem for classical Liénard equations of even degree

M. Caubergh, F. Dumortier *

Hasselt University, Campus Diepenbeek, Agoralaan, Gebouw D, 3590 Diepenbeek, Belgium

Received 11 May 2007; revised 13 November 2007

Available online 28 January 2008

Abstract

Classical Liénard equations are two-dimensional vector fields, on the phase plane or on the Liénard plane, related to scalar differential equations $\ddot{x} + f(x)\dot{x} + x = 0$. In this paper, we consider f to be a polynomial of degree $2l - 1$, with l a fixed but arbitrary natural number. The related Liénard equation is of degree $2l$. We prove that the number of limit cycles of such an equation is uniformly bounded, if we restrict f to some compact set of polynomials of degree exactly $2l - 1$. The main problem consists in studying the large amplitude limit cycles, of which we show that there are at most l.
© 2007 Elsevier Inc. All rights reserved.

MSC: 34C07; 34C37; 34D10

Keywords: Classical Liénard equation; Limit cycle; Heteroclinic connection; Cyclicity

1. Introduction

Hilbert's 16th problem [4] asks for the maximum number of limit cycles that a polynomial vector field, for a given degree, in the plane can have. Although the problem is more than 100 years old it is not even known whether a uniform upper bound, only depending on the degree of the vector field, might exist, even not when the degree is two. In the year 2000, S. Smale added the question to his list of problems for the 21st century [8], but restricting it to the classical (polynomial) Liénard equations.

* Corresponding author.
E-mail addresses: magdalena.caubergh@uhasselt.be (M. Caubergh), freddy.dumortier@uhasselt.be (F. Dumortier).

0022-0396/$ – see front matter © 2007 Elsevier Inc. All rights reserved.
doi:10.1016/j.jde.2007.11.011