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CYCLICITY OF UNBOUNDED SEMI-HYPERBOLIC 2-SADDLE CYCLES IN POLYNOMIAL LIENARD SYSTEMS

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Abstract. The paper deals with the cyclicity of unbounded semi–hyperbolic 2–saddle cycles in polynomial Liénard systems of type (m, n) with $m < 2n+1$, m and n odd. We generalize the results in [1] (case $m = 1$), providing a substantially simpler and more transparant proof than the one used in [1].

1. **Introduction.** In this paper we will study families of Liénard systems

$$(X_{(a,b)}) : \begin{cases} \dot{x} &= y - \left(x^{n+1} + \displaystyle\sum_{i=1}^{n} a_i x^{n+1-i}\right), \\ \dot{y} &= -\left(x^m + \displaystyle\sum_{i=1}^{m-1} b_i x^{m-i}\right), \end{cases} \tag{1}$$

with $(m, n) \in I\!N^2$, m and n odd and $m < 2n + 1$. We fix such (m, n). As an important ingredient of the construction, we observe that $X_{(a,b)}$ is invariant under

$$(x, y, a^o, a^e, b^o, b^e, t) \mapsto (-x, y, -a^o, a^e, -b^o, b^e, -t), \tag{2}$$

with $a^o = (a_1, a_3, \ldots, a_n)$, $a^e = (a_2, a_4, \ldots, a_{n-1})$, $b^o = (b_1, b_3, \ldots, b_{m-2})$ and $b^e = (b_2, b_4, \ldots, b_{m-1})$.

The motivation to study systems (1) comes from the scalar equations:

$$\ddot{x} + Q(x)\dot{x} + P(x) = 0, \tag{3}$$

with P and Q polynomials of respective strict degrees m and n and with the highest degree coefficient of P positive. In the phase plane equation (3) can be written as:

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= -P(x) - yQ(x). \end{cases} \tag{4}$$

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