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Generalized Liénard Equations, Cyclicity and Hopf–Takens Bifurcations

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We investigate the bifurcation of small–amplitude limit cycles in generalized Liénard equations. We use the simplicity of the Liénard family, to illustrate the advantages of the approach based on Bautin ideals. Essentially, this Bautin ideal is generated by the so–called Lyapunov quantities, that are computed for generalized Liénard equations and used to detect the presence of a Hopf–Takens bifurcation. Furthermore, the cyclicity is computed exactly.

Key Words: limit cycles, cyclicity, centers, Hopf bifurcations, Liénard equations, Lyapunov quantities.

1. INTRODUCTION

Computation of Lyapunov quantities is the key step for solving the stability problem of a plane system which is a perturbation of a linear focus at the origin. Recently many contributions have been devoted to analysing Bautin's approach to the local Hilbert's 16th problem. This involves algebraic techniques such as the Bautin ideal and bifurcation techniques such as normal forms.

There are many interests to consider Liénard equations (see for instance [6, 8, 7]). Furthermore, the (generalized) Liénard equation provides the