Chapter 4
Canard Cycles with Three Breaking Mechanisms

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Abstract This article deals with relaxation oscillations from a generic balanced canard cycle Γ subject to three breaking parameters of Hopf or jump type. We prove that in a rescaled layer of Γ there bifurcate at most five relaxation oscillations.

Keywords Balanced • n-multi-layer canard cycle • Breaking parameter • Rescaled layer • Cyclicity • Bifurcating limit cycle • Relaxation oscillations

4.1 Introduction

We consider slow fast systems of the form

$$X_{\lambda,\varepsilon}: \begin{cases} \dot{x} = f(x, y, \lambda, \varepsilon) \\ \dot{y} = \varepsilon g(x, y, \lambda, \varepsilon), \end{cases} \tag{4.1}$$

where f, g are smooth functions. In the study of relaxation oscillations we follow the general framework as introduced in [2, 3].

Each canard cycle is associated with one or more breaking mechanisms. As in [5] we consider only canard cycles with n generic breaking mechanisms, that may be Hopf breaking mechanisms and jump breaking mechanisms. Each mechanism depends on a so-called breaking parameter, in fact a function $a(\lambda)$ of the parameter λ. The assumed genericity is that the map $\lambda \to (a_1(\lambda), \ldots, a_n(\lambda))$ is a local diffeomorphism. Then, we will suppose that $\lambda = a = (a_1, \ldots, a_n)$. The canard cycle exists when $a = 0 \in \mathbb{R}^n$ and we want to study the system for $a \sim 0 \in \mathbb{R}^n$. A canard cycle with n breaking mechanisms is associated with n

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