International Journal of Bifurcation and Chaos, Vol. 23, No. 9 (2013) 1350161 (30 pages)

DOI: 10.1142/S0218127413501617

GLOBAL PHASE PORTRAITS OF SOME REVERSIBLE CUBIC CENTERS WITH NONCOLLINEAR SINGULARITIES

M. CAUBERGH* and J. TORREGROSA†
Departament de Matemàtiques,
Edifici C. 08193 Bellaterra, Barcelona, Spain
**leen@mat.uab.cat*
†torre@mat.uab.cat

Received April 4, 2013

The results in this paper show that the cubic vector fields $\dot{x} = -y + M(x, y) - y(x^2 + y^2)$, $\dot{y} = x + N(x, y) + x(x^2 + y^2)$, where M, N are quadratic homogeneous polynomials, having simultaneously a center at the origin and at infinity, have at least 61 and at most 68 topologically different phase portraits. To this end, the reversible subfamily defined by $M(x, y) = -\gamma xy$, $N(x, y) = (\gamma - \lambda)x^2 + \alpha^2 \lambda y^2$ with $\alpha, \gamma \in \mathbb{R}$ and $\lambda \neq 0$, is studied in detail and it is shown to have at least 48 and at most 55 topologically different phase portraits. In particular, there are exactly five for $\gamma\lambda < 0$ and at least 46 for $\gamma\lambda > 0$. Furthermore, the global bifurcation diagram is analyzed.

Keywords: Reversible planar vector field; cubic vector field; global classification of phase portraits; bifurcation diagram.

1. Introduction

This paper completes the topological classification of the global phase portraits on the Poincaré disc of the six-parameter family of cubic differential equations

$$\begin{aligned} \dot{x} &= -y + ax^2 + bxy + cy^2 - y(x^2 + y^2), \\ \dot{y} &= x + ex^2 + fxy + gy^2 + x(x^2 + y^2), \end{aligned} \tag{1}$$

that have simultaneously a center in the origin and at infinity for $a, b, c, e, f, g \in \mathbb{R}$. This study was started in [Caubergh *et al.*, 2011, 2012].

Recently related analysis are done in [Artés *et al.*, 2006, 2010; Artés *et al.*, 2013; Cao & Jiang, 2008; Li & Wang, 2011; Oliveira & Rezende, 2013]. In particular, in [Oliveira & Rezende, 2013], the so-called SIS-model is considered, that is used in the study of infectious diseases. The papers [Artés *et al.*, 2006, 2010; Artés *et al.*, 2013; Cao & Jiang, 2008; Oliveira & Rezende, 2013] contribute to the classification of planar quadratic differential systems; due to the six-dimensional parameter and the richness of phase portraits, its bifurcation diagram is also studied for intrinsic subclasses reducing its dimension, and similarly these sub-bifurcation diagrams are then analyzed by slicing and imbedding in projective planes. Furthermore, another generalization can be found in [Li & Wang, 2011], where a global topological classification is studied for a one-parameter cubic Hamiltonian planar differential system in which a finite center is linked to singularities at infinity.

In [Blows & Rousseau, 1993] the differential systems (1) are characterized by a Hamiltonian class and a reversible class, that is symmetric with respective to straight lines. In [Caubergh *et al.*, 2011, 2012] respectively the classification is obtained for the full Hamiltonian class and part of the reversible class, i.e. the ones having infinitely many singularities or all singularities on the line of