PERIODIC SOLUTIONS AND THEIR STABILITY OF SOME HIGHER-ORDER POSITIVELY HOMOGENOUS DIFFERENTIAL EQUATIONS

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Abstract. In the present paper we study periodic solutions and their stability of the m-order differential equations of the form

$$x^{(m)} + f_n(x) = \mu h(t),$$

where the integers $m, n \geq 2$, $f_n(x) = \delta x^n$ or $\delta |x|^n$ with $\delta = \pm 1$, and $h(t)$ is a continuous T-periodic function of non-zero average, and μ is a positive small parameter. By using the averaging theory, we will give the existence of T-periodic solutions. Moreover, the instability and the linear stability of these periodic solutions will be obtained.

1. Introduction and statement of the main results

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In this paper we are concerned with periodic solutions of some higher order homogenous or positively homogenous differential equations. A typical example is the second-order ordinary differential equation

$$\ddot{x} + x^3 = h(t). \qquad (1)$$ e1

In [10] Morris proved that if $h(t)$ is a T-periodic C^1 function and its average

$$\bar{h} := \frac{1}{T} \int_0^T h(t) dt$$

is zero, then Eq. (1) has periodic solutions of period kT for all positive integer k. Later on the same result was proved by Ding and Zanolin [5] without the assumption that $\bar{h} = 0$. More recently Ortega in [11] proved that Eq. (1) has finitely many stable periodic solutions of a fixed period.

Other authors have studied more general problems related with non-autonomous differential equations, as for instance: when a periodic

2010 *Mathematics Subject Classification.* 37G15, 37C80, 37C30.

Key words and phrases. periodic solution, m-order differential equations, stability, averaging theory.