Minimum periods of homeomorphisms of orientable surfaces

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One of the main problems of the theory of dynamical systems is the determination of the existence of periodic orbits of a selfmap and more generally, the structure of the set of periods. In connection with this structure, we define the minimum period of a class os selfmaps of a fixed set as the minimum of the positive integers such that each map in the class has a periodic point whose period is at most this number.

In the case of closed surfaces, the problem of the determination of the minimum period of the classes of homeomorphisms was completely solved, in successive steps, from 1910 to 1996. The aim of our work is, for each compact, connected, orientable surface, determine the minimum period of its class of homeomorphisms.

If the genus of the considered surface is 0 or 1, then the problem can be solved by simple techniques. For the case of genus at least 2, we have found two upper bounds for the minimum periods, which can be expressed as a linear function of the genus and the number of boundary components of the surface. We give certain suficient conditions under which these upper bounds are achieved. In particular, we have proved that the minimum period becomes constant for each genus, provided that the number of boundary components is large enough. We have also studied the minimum periods of the classes of finite-order maps.

This thesis has three branches which are interconneced. One has to do with the application of the fixed-point theory. One of the upper bounds of the minimum periods is a consequence of this theory. To obtain the other upper bound, we have also applied the Thurston-Nielsen classification of homeomorphisms of surfaces and some of its consequences. This is the second branch. Finally, the third branch has to do with the theory of planar discontinuous group which provide us with the necessary tools for the construction of examples which prove the existence of lower bounds of the minimum periods.