Isochronous Centers of Cubic Reversible Systems

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Abstract. In this paper we study isochronous centers of reversible two-dimensional autonomous system with linear part of center type and nonlinear part given by polynomials of third degree. Firstly we find necessary conditions for such isochronous centers in polar coordinates and finally we give a proof of the isochronicity of these systems using different methods.

1 Introduction

It is known that the problem of isochronicity appears only for *nondegenerate centers*, see Mardesic et al. (1995).

We consider cubic polynomial systems with a nondegenerate center at the origin. In an appropriate coordinate system and upon rescaling of the independent variable these systems take the form

$$\dot{x} = -y + X_2(x,y) + X_3(x,y), \quad \dot{y} = x + Y_2(x,y) + Y_3(x,y), \tag{1}$$

where $\dot{} = \mathrm{d}/\mathrm{d}\,t$ and $X_s(x,y)$ and $Y_s(x,y)$, $s = 2,3$, are *homogeneous polynomials* of degree s. We say that (1) is *reversible* if it is invariant under a rotation

$$\begin{pmatrix} \xi \\ \eta \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$$

and a time inversion $t = -\tau$. Reversible systems are characterised by the existence of at least one straight line containing the origin, which is an axis of symmetry of the phase portrait. This line is given by the angle $\alpha/2$.

The integrable cases for homogeneous systems of the form

$$\dot{x} = -y + X_s(x,y), \quad \dot{y} = x + Y_s(x,y),$$

have been studied by several authors, namely *quadratic systems*, $s = 2$, and *cubic homogeneous systems*, $s = 3$, by Bautin (1952), Chavarriga (1994), Coppel (1996), Lloyd (1983), Lunkevich and Sibirskii (1982), Schlomiuk (1993) and Zoladek (1994). Some integrable cases of homogeneous systems when $s = 4,5$ have been determined by Chavarriga and Giné (1996), Chavarriga and Giné (1997).