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Integrability and Explicit Solutions in Some Bianchi Cosmological Dynamical Systems

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Abstract

The Einstein field equations for several cosmological models reduce to polynomial systems of ordinary differential equations. In this paper we shall concentrate our attention to the spatially homogeneous diagonal G_2 cosmologies. By using Darboux's theory in order to study ordinary differential equations in the complex projective plane $\mathbb{CP}^2$ we solve the Bianchi V models totally. Moreover, we carry out a study of Bianchi VI models and first integrals are given in particular cases.

1 Introduction

The Einstein field equations for a number of classes of cosmological models have been written as polynomial systems of ordinary differential equations. In recent investigations some authors look for new exact solutions of these differential systems due to their cosmological interest. Most of these solutions araise either due to the existence of a symmetry, or under a restriction on the parameters of the cosmological model. Homogeneous cosmological models have important applications in the theory of evolution of the universe. In particular they can be applied to the theory of explosions of stars, formation of galaxies, etc. The equations of general relativity are reduced to systems of a finite (but quite large) number of ordinary differential equations. In the last two decades these dynamical systems were and still are being analyzed by means of traditionals analytic and numerical methods. However the dynamical systems in question are so complex and the development of effective methods for their analysis becomes especially urgent.

In this paper we shall concentrate our attention to the diagonal G_2 cosmologies, i.e. those spacetimes which admit an Abelian G_2 comprising two hypersurface-orthogonal Killing vector fields (see for instance [5]). In fact we only study spatially homogeneous diagonal G_2 cosmologies. It is assumed that the Einstein field equations are satisfied and that spacetime admits a perfect fluid with linear equation of state

$$p = (\gamma - 1)\rho, \qquad \frac{2}{3} < \gamma < 2, \tag{1}$$