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ON INTEGRABILITY OF DIFFERENTIAL EQUATIONS DEFINED BY THE SUM OF HOMOGENEOUS VECTOR FIELDS WITH DEGENERATE INFINITY

J. CHAVARRIGA*, I. A. GARCÍA† and J. GINÉ‡
Departament de Matemàtica, Universitat de Lleida
Avda. Jaume II, 69. 25001 Lleida, Spain

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The paper deals with polynomials systems with degenerate infinity from different points of view. We show the utility of the projective techniques for such systems, and a more detailed study in the quadratic and cubic cases is carried out. On the other hand, some results on Darboux integrability in the affine plane for a class of systems are given. In short we show the explicit form of generalized Darboux inverse integrating factors for the above kind of systems. Finally, a short proof of the center cases for arbitrary degree homogeneous systems with degenerate infinity is given, and moreover we solve the center problem for quartic systems with degenerate infinity and constant angular speed.

1. Introduction

Throughout this paper we will consider planar polynomial autonomous differential systems with degenerate infinity, i.e. systems of the form

$$\begin{aligned} \dot{x} &= P(x,\, y) = \sum_{k=0}^{m} P_k(x,\, y)\,, \\ \dot{y} &= Q(x,\, y) = \sum_{k=0}^{m} Q_k(x,\, y)\,, \end{aligned} \tag{1}$$

where the over dot denotes as usual the derivative with respect to the independent variable t, and P_k and Q_k are real homogeneous polynomials of degree k. Moreover, the polynomials P_m and Q_m satisfy the *degenerate infinity* condition

$$xQ_m(x,\, y) - yP_m(x,\, y) \equiv 0\,. \tag{2}$$

The name degenerate infinity is due to the fact that, working with the *homogeneous coordinates* $(X,\, Y,\, Z)$ of the complex projective plane $\mathbb{CP}^2$, system (1) has the invariant line of infinity $Z = 0$ filled up of critical points.

Degenerate infinity systems have attracted the attention of many authors. For instance, [Gasull & Prohens, 1996] give an affine classification of quadratic systems. A subfamily of such quadratic systems is studied in [Chen & Liang, 1993]. Yasmin [1989] calculates the Liapunov quantities for system (1), for $m = 3$ and with the linear part of center type, i.e. with $P_0 = Q_0 = 0$, $P_1 = -y$ and $Q_1 = x$, in order to find the maximum order of the origin, and hence the number of limit cycles that can be produced from it by bifurcation. Christopher [1994] gives local first integrals or integrating factors for these types of cubic systems. Chavarriga and Giné [1998] show, that for all cubic

*The author is partially supported by a DGICYT grant, number PB96-1153. E-mail: chava@eup.udl.es
†The author is partially supported by a University of Lleida Project P98-207. E-mail: garcia@eup.udl.es
‡The author is partially supported by a University of Lleida Project P98-207. E-mail: gine@eup.udl.es