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On Lie's symmetries for planar polynomial differential systems

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Abstract

In this paper we show that any polynomial planar vector field which is either polynomial or rational integrable possesses a polynomial or rational infinitesimal generator of a Lie symmetry, respectively. Moreover, if all the critical points of the vector field are strong and there exists a polynomial inverse integrating factor that vanishes at all the critical points we show that, independently of the class of their first integral, there exists a polynomial infinitesimal generator of a Lie symmetry.

Mathematics Subject Classification: 58G35, 22E05, 34A34, 58F22

1. Introduction

One of the most appealing applications of Lie's group theory is to the problem of integrating ordinary differential equations. The fundamental observation of Lie was that knowledge of a sufficiently large group of symmetries of a system of ordinary differential equations allows one to integrate the system by quadratures. This approach unifies and significantly extends the various special methods introduced for the integration of certain types of first-order equations such as separable, homogeneous, exact and so on.

This paper introduces an approach to important questions related to first integrals, in the spirit of the great geometrical approach initiated by Hamilton, Poincaré and Noether, in the algebraic case. Similarly to the famous theorem of Noether which establishes a duality between the first integrals of a mechanical system and its infinitesimal symmetries, one has a correspondence between the inverse integrating factor and the commuting vector fields in the Lie sense, see proposition 9. We investigate the existence and the class of Lie symmetries for planar polynomial differential systems. The fundamental idea of this work consists on demonstrating the existence of infinitesimal generators of Lie symmetries of the same functional class as either the inverse integrating factor or the first integral. The proofs are based on two classical results of algebraic geometry, namely Hilbert's zero theorem and Nöether's theorem.

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