

Polynomial first integrals of quadratic vector fields

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Abstract

We classify all quadratic polynomial differential systems having a polynomial first integral, and provide explicit normal forms for such systems and for their first integrals.

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1. Introduction and statement of the results

Let $\mathbb{R}[x, y]$ be the ring of all polynomials in the variables x and y with coefficients in $\mathbb{R}$. In this paper we deal with *quadratic polynomial differential systems* in $\mathbb{R}^2$ of the form

$$\frac{dx}{dt} = x' = P(x, y), \quad \frac{dy}{dt} = y' = Q(x, y), \quad (1)$$

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