

Invariant algebraic curves for the cubic Liénard system with linear damping [☆]

J. Chavarriga ^{a,✉}, I.A. García ^a, J. Llibre ^b, H. Żołądek ^{c,*}

^a *Departament de Matemàtica, Universitat de Lleida, Av. Jaume II, 69, 25001 Lleida, Spain*

^b *Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*

^c *Institute of Mathematics, University of Warsaw, ul. Banacha 2, 02-097 Warsaw, Poland*

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Abstract

We show that the system $\dot{x} = y$, $\dot{y} = -f(x)y - g(x)$ with f, g polynomials of degree 1 and 3 respectively cannot have simultaneously an algebraic invariant curve and a limit cycle.

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1. Introduction

In 1995 K. Odani [10] started the study of invariant algebraic curves for the Liénard system

$$\dot{x} = y, \quad \dot{y} = -f_m(x)y - g_n(x), \quad f_m g_n \not\equiv 0, \quad (1)$$

where f_m and g_n are polynomials of degree m and n respectively. He has shown that the system (1) does not have invariant algebraic curves when $n \leq m$ and $g_n/f_m \not\equiv \text{constant}$. In particular, the limit cycle of the van der Pol system (i.e. for $m = 2$ and $n = 1$) is not algebraic.

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^{*} Corresponding author.

E-mail addresses: chava@eps.udl.es (J. Chavarriga), garcia@matemtica.udl.es (I.A. García), jllibre@mat.uab.es (J. Llibre), zoladek@mimuw.edu.pl (H. Żołądek).

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