



An Improvement to Darboux Integrability Theorem for Systems Having a Center

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Abstract—We consider planar polynomial differential systems of degree m with a center at the origin and with an arbitrary linear part. We show that if the system has $m(m+1)/2 - [(m+1)/2]$ algebraic solutions or exponential factors then it has a Darboux integrating factor. This result is an improvement of the classical Darboux integrability theorem and other recent results about integrability. © 1999 Elsevier Science Ltd. All rights reserved.

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1. INTRODUCTION

This work contributes to show the links between the theory of polynomial systems and the algebraic curves. Indeed, already in 1878, Darboux [1] showed how the first integrals of polynomial systems possessing sufficient algebraic solutions are constructed. In particular, he proved that if a polynomial system of degree m has at least $m(m+1)/2$ algebraic solutions, then it has a first integral or an integrating factor. On the other hand, such links were also suggested in 1900 by the way that Hilbert in [2] stated his 16th problem into two parts: the first one about the topology of real algebraic curves and the second part about the maximum number of limit cycles of polynomial systems having a given degree. Recently, these links appeared in the center problem for quadratic systems, see [3,4].

This paper first presents a short survey about the Darboux method in Section 2. As a matter of fact the best two improvements to the Darboux Theorem are due to Jouanolou [5] in 1979 and Prelle and Singer [6] in 1983. The first, showing that if the number of algebraic solutions of a