JOURNAL OF DIFFERENTIAL EQUATIONS **157**, 163–182 (1999)
Article ID jdeq.1998.3621, available online at http://www.idealibrary.com on IDEAL®

On the Integrability of Two-Dimensional Flows

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Received June 10, 1998; revised November 5, 1998

This paper deals with the notion of integrability of flows or vector fields on two-dimensional manifolds. We consider the following two key points about first integrals: (1) They must be defined on the whole domain of definition of the flow or vector field, or defined on the complement of some special orbits of the system; (2) How are they computed? We prove that every local flow φ on a two-dimensional manifold M always has a continuous first integral on each component of $M\backslash\Sigma$ where Σ is the set of all separatrices of φ. We consider the inverse integrating factor and we show that it is better to work with it instead of working directly with a first integral or an integrating factor for studying the integrability of a given two-dimensional flow or vector field. Finally, we prove the existence and uniqueness of an analytic inverse integrating factor in a neighborhood of a strong focus, of a non-resonant hyperbolic node, and of a Siegel hyperbolic saddle.

Key Words: First integral; two-dimensional differential systems; inverse integrating factor.

0022-0396/99 $30.00