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Local analytic integrability for nilpotent centers

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Abstract. Let $X(x, y)$ and $Y(x, y)$ be real analytic functions without constant and linear terms defined in a neighborhood of the origin. Assume that the analytic differential system $\dot{x} = y + X(x, y)$, $\dot{y} = Y(x, y)$ has a nilpotent center at the origin. The first integrals, formal or analytic, will be real except if we say explicitly the converse. We prove the following.

(a) If $X = yf(x, y^2)$ and $Y = g(x, y^2)$, then the system has a local analytic first integral of the form $H = y^2 + F(x, y)$, where F starts with terms of order higher than two.

(b) If the system has a formal first integral, then it has a formal first integral of the form $H = y^2 + F(x, y)$, where F starts with terms of order higher than two. In particular, if the system has a local analytic first integral defined at the origin, then it has a local analytic first integral of the form $H = y^2 + F(x, y)$, where F starts with terms of order higher than two.

(c) As an application we characterize the nilpotent centers for the differential systems $\dot{x} = y + P_3(x, y)$, $\dot{y} = Q_3(x, y)$, which have a local analytic first integral, where P_3 and Q_3 are homogeneous polynomials of degree three.

1. *Introduction and statement of the main results*

One of the more classical problems in the qualitative theory of planar real analytic differential systems is to characterize the local phase portrait at an isolated singular point. This problem has been solved except if the singular point is a center or a focus,