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Darboux integrability and the inverse integrating factor ☆

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Abstract

We mainly study polynomial differential systems of the form $dx/dt = P(x,y)$, $dy/dt = Q(x,y)$, where P and Q are complex polynomials in the dependent complex variables x and y, and the independent variable t is either real or complex. We assume that the polynomials P and Q are relatively prime and that the differential system has a Darboux first integral of the form

$$H = f_1^{\lambda_1} \cdots f_p^{\lambda_p} \left(\exp\left(\frac{h_1}{g_1^{n_1}}\right)\right)^{\mu_1} \cdots \left(\exp\left(\frac{h_q}{g_q^{n_q}}\right)\right)^{\mu_q},$$

where the polynomials f_i and g_j are irreducible, the polynomials g_j and h_j are coprime, and the λ_i and μ_j are complex numbers, when $i = 1, \ldots, p$ and $j = 1, \ldots, q$. Prelle and Singer proved that these systems have a rational integrating factor. We improve this result as follows.

Assume that H is a rational function which is not polynomial. Following to Poincaré we define the critical remarkable values of H. Then, we prove that the system has a polynomial inverse integrating factor if and only if H has at most two critical remarkable values.

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