QUADRATIC SYSTEMS WITH AN ALGEBRAIC LIMIT CYCLE OF DEGREE 2 OR 4 DO NOT HAVE A LIOUVILLIAN FIRST INTEGRAL*

J. CHAVARRIGA [(1)], H. GIACOMINI [(2)] AND M. GRAU [(1)]

[(1)] *Departament de Matemàtica. Universitat de Lleida.*
Avda. Jaume II, 69. 25001 Lleida, SPAIN.
E–mails: chava@eup.udl.es and mtgrau@matematica.udl.es

[(2)] *Laboratoire de Mathématiques et Physique Théorique.*
CNRS UMR 6083.
Faculté des Sciences et Techniques. Université de Tours.
Parc de Grandmont 37200 Tours, FRANCE.
E-mail: giacomin@celphi.phys.univ-tours.fr

We consider the families of quadratic systems in the projective plane with algebraic limit cycles of degree 2 or 4. There are no algebraic limit cycles of degree 3 for a quadratic system. Until the moment, no other families of quadratic systems with an algebraic limit cycle, not birrationally equivalent to the ones that we study, have been found. We prove that none of these systems has a Liouvillian first integral. Our main tool is the characterization of the form of the cofactor of an irreducible invariant algebraic curve, when this curve exists, by means of the study of the singular points of the system. For obtaining this characterization of the form of the cofactor we consider the behavior of the solutions of the system in a neighborhood of a critical point.

1. Statement of the result

We consider a *planar polynomial differential system*

$$\dot{x} = P(x,y), \quad \dot{y} = Q(x,y), \tag{1}$$

where $P(x,y), Q(x,y) \in \mathbb{R}[x,y]$ are coprime polynomials. Let $d = \max\{\deg P, \deg Q\}$ be the *degree* of system (1). We say that the system is *quadratic* when $d = 2$.

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