that $g(x, y_h(u_h)(x)) \leq \delta \; \forall x \in \bar{\Omega}$ and $\forall h \leq h^j$. Hence u_h is a feasible control for $(\mathrm{P}_{\delta h})$ always that $h \leq h^j$. Then we get $J_{h_k R}(\bar{r}_{h_k}) = J_{h_k}(\bar{u}_{h_k}) \leq J_{h_k}(u_{h_k})$ whenever $h_k \leq h^j$. Thus we have

$$J_R(\bar{r}) = \lim_{k\to\infty} J_{h_k}(\bar{r}_{h_k}) \leq \lim_{k\to\infty} J_{h_k}(u_{h_k}) = J(u^j).$$

Now taking the limit when $j \to \infty$, we obtain that $J_R(\bar{r}) \leq J_R(r_{\delta'})$. Finally, the feasibility of $\bar{r}$ for (RP_δ) and the stability condition (Definition 2.2) enables us to conclude that

$$\inf(\mathrm{RP}_\delta) \leq J_R(\bar{r}) \leq \lim_{\delta' \nearrow \delta} J_R(r_{\delta'}) = \lim_{\delta' \nearrow \delta} \inf(\mathrm{RP}_{\delta'}) \leq$$

$$\lim_{\delta' \nearrow \delta} \inf(\mathrm{P}_{\delta'}) = \inf(\mathrm{P}_\delta) = \inf(\mathrm{RP}_\delta),$$

which proves that $\bar{r}$ is a solution of (RP_δ). The rest of the theorem is immediate.

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INTEGRABILITY OF A LINEAR CENTER PERTURBED BY HOMOGENEOUS POLYNOMIAL

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Abstract. Consider in the plane the ordinary differential equation system of the form

$$(1) \qquad \dot{x} = \lambda x - y + X(x,y), \; \dot{y} = x + \lambda y + Y(x,y), \quad \dot{} = \frac{d}{dt},$$

being $X(x,y)$ and $Y(x,y)$ analytical functions, definite in a certain domain around the origin and which do not have linear and constant terms in their power series development. Our objective is the research of the integrability of these systems.

1. Historical antecedents

In the beginning of the century, Hilbert [10] in the International Mathematics Congress, held in Paris in the year 1900, formulated some questions that he though that its solution is fundamental in the development of the mathematics. In particular he enunciated, among others, the one called 16 problem about the topology research of real algebraic manifolds and in its b) section he talks about the "maximum number of limit cycles of Poincaré for a first order and first degree ordinary differential equation of the form $\frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$ where $M(x,y)$ and $N(x,y)$ are polynomials of degree n in the x and y variables.

The problem starts with the first description that Poincaré [17] did about the phenomenon of the appearance of one limit cycle in one ordinary differential equation system. Later it was enunciated