Publicacions Matemàtiques, Vol **40** (1996), 21–39.

INTEGRABILITY OF A LINEAR CENTER PERTURBED BY A FOURTH DEGREE HOMOGENEOUS POLYNOMIAL*

Javier Chavarriga and Jaume Giné

Abstract

In this work we study the integrability of a two-dimensional autonomous system in the plane with linear part of center type and non-linear part given by homogeneous polynomials of fourth degree. We give sufficient conditions for integrability in polar coordinates. Finally we establish a conjecture about the independence of the two classes of parameters which appear in the system; if this conjecture is true the integrable cases found will be the only possible ones.

1. Introduction

We consider the system

$$\begin{aligned} \dot{x} &= -y + X_s(x,y), \\ \dot{y} &= x + Y_s(x,y), \end{aligned} \tag{1.1}$$

where $X_s(x,y)$ and $Y_s(x,y)$ are homogeneous polynomials of degree s, with $s \geq 2$.

The aim of this paper is to find the integrable cases of system (1.1) when $s = 4$ (see Theorem 1). The integrable cases for quadratic systems, $s = 2$, and cubic homogeneous systems, $s = 3$, have been studied by several authors: Bautin [**1**], Chavarriga [**2**], Coppel [**5**], Frommer [**6**], Kapteyn [**7**], Lloyd [**8**], Lunkevich and Sibirskii [**9**], Schlomiuk [**11**] and Żoladek [**15**]. Poincaré [**10**] developed an important technique for the general solution of these problems. It consists in finding a formal power series of the form

$$H(x,y) = \sum_{n=2}^{\infty} H_n(x,y), \tag{1.2}$$

*Research partially supported by a University of Lleida Project/94.