INTEGRABILITY OF A LINEAR CENTER PERTURBED BY CUBIC POLYNOMIALS

J. CHAVARRIGA AND J. GINÉ

Departament de Matemàtica. Universitat de Lleida. Pl. Victor Siurana, 1. 25003 Lleida, SPAIN.

Introduction

The objective of this work is to present some sufficient and necessary conditions for the integrability of differential equation systems in the plane with a linear center and with cubic polynomial nonlinearities.

We consider the ordinary differential equation system in the plane of the form

$$\dot{x} = -y + X_3(x,y) \ , \quad \dot{y} = x + Y_3(x,y) \ , \quad \dot{} = \frac{d}{dt} \ , \tag{1}$$

where $X_3(x,y)$ and $Y_3(x,y)$ are third degree polynomials without constants and linear terms. We will call this system *cubic complete system.*

The problem of distinguishing between a center and a focus is called *the cubic center-focus problem.* The first step to solving this problem is the calculation of the Lyapunov constants. This is a computational problem because we are dealing with polynomials with long rational coefficients. There are diverse methods of the computation of Lyapunov constants or the focus quantities, see for example [4], [5], [7], [9] and [10]. When this problem is resolved we have to find a basis for the ideal generated by these constants that is finitely generated applying Hilbert's theorem. There exists an algorithmical method to solve such kind of problems, the Gröbner basis theory, but it is only applicable for simple cases. All these methods are used to obtain the necessary conditions for a center, but to ensure that these are sufficient it is necessary to find an analytic first integral or an integrating factor of the system. In this case we say that we have found the integrable cases. Several authors have found integrable cases for system (1) see for example [3], [6], [7], [8], [11], [13] and [14].