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Integrable Systems Via Inverse Integrating Factor

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1. Introduction

We consider two-dimensional autonomous systems of differential equations of the form

$$\dot{x} = -y + X_s(x,y), \quad \dot{y} = x + Y_s(x,y), \tag{1}$$

where

$$X_s(x,y) = \sum_{k=0}^{s} a_k x^k y^{s-k}, \quad Y_s(x,y) = \sum_{k=0}^{s} b_k x^k y^{s-k},$$

are homogeneous polynomials of degree s, with $s \geq 2$; being a_k and b_k, $k = 0, 1, \ldots, s$, arbitrary real coefficients. Recently, these systems have been studied by several authors (see for instance [1], [2], [4], [5], [11] and [12]), especially in order to obtain information about the number of small amplitude limit cycles and to determine the cyclicity of the origin (see for instance [1] and [12]). Our aim is to find solutions $V(x,y) = 0$ of system (1) where $V(x,y)$ is an inverse integrating factor (this notion will be defined below). The method consists in characterizing the systems which have an inverse integrating factor. This paper contains a natural generalization of the results developed in [3]. Theorem 1 gives an explicit method for obtaining such inverse integrating factor, which is used in Theorem 2 to construct some particular class of integrable vector fields. The method shows that if $V(x,y)$ is a product of n linear factors, elevated each factor to α_i, we always arrive to a partial differential

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