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POLYNOMIAL FIRST INTEGRALS OF SYSTEMS IN THE PLANE WITH CENTER TYPE LINEAR PART†

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1. INTRODUCTION

We consider two-dimensional autonomous systems of differential equations of the form

$$\begin{aligned}\dot{x} &= -y + X(x, y), \\ \dot{y} &= x + Y(x, y),\end{aligned}\tag{1}$$

where $X(x, y)$ and $Y(x, y)$ are analytic functions defined in certain neighbourhood of the origin without linear terms. Poincaré developed a valid technique for the determination of the first integral of these systems: it consists in finding a formal power series of the form

$$H(x, y) = \sum_{n=2}^{\infty} H_n(x, y),\tag{2}$$

where $H_2(x, y) = (x^2 + y^2)/2$, $H_n(x, y)$ being homogeneous polynomials of degree n , in such a way that

$$\dot{H} = \sum_{k=2}^{\infty} V_{2k}(x^2 + y^2)^k,\tag{3}$$

where V_{2k} are real numbers called *Lyapunov constants*. The vanishing of all Lyapunov constants is a necessary condition for the integrability of system (1), in this case the considered series $H(x, y)$ will be a first integral of the system if it is convergent, an open question today. On the other hand, it will not always be possible to express this first integral by means of elementary functions.

The aim of this paper consists in giving a characterisation of some systems (1) which have a polynomial first integral. In particular, we will study the systems where the functions $X(x, y)$ and $Y(x, y)$ are homogeneous polynomials of the same degree. Systems (1) which have a polynomial first integral have a center at the origin, see Lyapunov [1]. Other authors which study the existence of analytic integrals for a polynomial systems are Almukhamedov [2] and Otrokov [3]. The reasons to determine the systems which have a polynomial first integral are: the simplicity of their solutions and to obtain systems where we can apply the methods developed for Shi Songling [4] and [5] to determine

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