Local Integrability for Nilpotent Critical Point

Javier Chavarriga and Jaume Giné

Departament de Matemàtica, Universitat de Lleida, Pl. Victor Siurana 1, 25003, Lleida, Spain

Abstract. We study the two-dimensional autonomous systems of differential equations of the form $\dot{x} = y + a_{20}x^2 + a_{11}xy + a_{02}y^2, \dot{y} = b_{20}x^2 + b_{11}xy + b_{02}y^2$, where a_{ij} and b_{ij} are arbitrary real constants. The origin is a degenerate critical point of this system. In this work we give a sufficient conditions for local integrability at the origin.

1 Introduction

We consider two-dimensional autonomous systems of differential equations of the form

$$\dot{x} = y + a_{20}x^2 + a_{11}xy + a_{02}y^2, \quad \dot{y} = b_{20}x^2 + b_{11}xy + b_{02}y^2, \tag{1}$$

where a_{ij} and b_{ij} are arbitrary real coefficients.

These systems are particular cases of systems of the form

$$\dot{x} = y + F(x,y), \quad \dot{y} = G(x,y), \tag{2}$$

where F and G are analytic functions in a neighborhood of the origin starting in quadratic terms in the variables x and y. Systems (2) have been studied by several authors (see for instance Andreev 1958; Moussu 1982; Takens 1974 and Żoladek 1996).

Takens (1974) proves that the system (2) can be formally reduced to

$$\dot{x} = y + a(x), \quad \dot{y} = b(x), \tag{3}$$

where $a(x) = a_r x^r + a_{r+1}x^{r+1} + \ldots$ and $b(x) = b_{s-1}x^{s-1} + b_s x^s + \ldots$ are formal power series.

Bogdanov and some years later Żoladek (1996) proved that the system (3) reduces to the form

$$\dot{x} = y + a(x), \quad \dot{y} = x^{s-1}. \tag{4}$$

In this last work Żoladek has shown that the normal forms (3) and (4) can be chosen analytic.

Moussu (1982) finds necessary and sufficient conditions to have a center at the origin for systems (3). His result is the following: