

Journal of Computational and Applied Mathematics 126 (2000) 351-368

JOURNAL OF COMPUTATIONAL AND APPLIED MATHEMATICS

www.elsevier.nl/locate/cam

Isochronous centers of a linear center perturbed by fifth degree homogeneous polynomials

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Received 3 August 1998; received in revised form 17 September 1999

Abstract

In this work we study isochronous centers of two-dimensional autonomous system in the plane with linear part of center type and nonlinear part given by homogeneous polynomials of fifth degree. A complete classification of the necessary conditions for the time-reversible systems of this class is given in order to have an isochronous center at the origin. An open problem is stated for the sufficient conditions. Moreover, we find two nonreversible isochronous families from the center cases known. All the computations in order to obtain necessary conditions for such isochronous centers are given in polar coordinates and we give a proof of the isochronicity of these systems by using different methods. © 2000 Elsevier Science B.V. All rights reserved.

MSC: 34A05; 34C05

Keywords: Isochronicity; Center-focus problem; Nonlinear differential equations

1. Introduction

We consider here the isochronicity problem for a class of homogeneous plane polynomial differential system. It is known, see [14], that this problem appears only for *nondegenerate centers*, i.e., centers whose linear part has nonzero imaginary eigenvalues. Since these systems have a nonzero linear part at the center, we can take, without loss of generality, an appropriate coordinate system (x, y) and rescaling of the independent variable t in order to write the polynomial differential system as a linear center perturbed by homogeneous polynomials, i.e.,

$$\dot{x} = -y + X_s(x, y), \qquad \dot{y} = x + Y_s(x, y),$$
(1)

where $\cdot = d/dt$, $X_s(x, y)$ and $Y_s(x, y)$ being homogeneous polynomials of degree s, with $s \ge 2$.

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