ON THE ALGEBRAIC LIMIT CYCLES OF QUADRATIC SYSTEMS

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ABSTRACT

We study the algebraic limit cycles of quadratic polynomial differential systems in the plane. It is known that for such systems there are algebraic limit cycles of degree 2 and 4, and that there is no algebraic limit cycles of degree 3. We present new and shorter proofs of these last two results. We conjecture that these systems have no algebraic limit cycles of degree larger than 4.

1. Introduction

We consider a system of differential equations of the form

$$\begin{aligned} \dot{x} &= a_{00} + a_{10}x + a_{01}y + a_{10}x^2 + a_{11}xy + a_{02}y^2, \\ \dot{y} &= b_{00} + b_{10}x + b_{01}y + b_{10}x^2 + b_{11}xy + b_{02}y^2, \end{aligned} \qquad (1)$$

where a_{ij} and b_{ij} are real numbers, i.e. a *quadratic system.*

We say that (x_0, y_0) is a *singular point* of system (1) if it satisfies that $\dot{x}(x_0, y_0) = \dot{y}(x_0, y_0) = 0$. A *limit cycle* of system (1) is an isolated periodic solution in the set of all periodic solutions.

The aim of this paper is to characterize the quadratic systems (1) which possess an *algebraic limit cycle* of degree 2, 3 or 4; i.e., algebraic curves $f(x, y) = 0$ which are particular solutions of system (1) containing a real closed oval which is a limit cycle, where $f(x, y) = 0$ is an irreducible polynomial of degree 2, 3 or 4, respectively.

For quadratic algebraic curves $f(x, y) = 0$, the following result is known, see [7] or [10].

Theorem 1.~~4~~1 *If system (1) has an algebraic limit cycle of degree 2, then after an affine change of variables the limit cycle becomes the circle* $\Gamma := x^2 + y^2 - 1 =$