Journal of Differential Equations **169**, 1–16 (2001)
doi:10.1006/jdeq.2000.3891, available online at http://www.idealibrary.com on IDEAL®

Invariant Algebraic Curves and Rational First Integrals for Planar Polynomial Vector Fields[1]

Javier Chavarriga

Departament de Matemàtica, Universitat de Lleida, Avda. Jaume II, 69, 25001 Lleida, Spain
E-mail: chava@eup.udl.es

and

Jaume Llibre

Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193-Bellaterra, Barcelona, Spain
E-mail: jllibre@mat.uab.es

Received April 2, 1999; revised September 28, 1999

DEDICATED TO PROFESSOR JACK K. HALE ON THE OCCASION OF HIS 70TH BIRTHDAY

We present three main results. The first two provide sufficient conditions in order that a planar polynomial vector field in $\mathbf{C}^2$ has a rational first integral, and the third one studies the number of multiple points that an invariant algebraic curve of degree n of a planar polynomial vector field of degree m can have in function of m and n. © 2001 Academic Press

1. INTRODUCTION

By definition a *complex planar polynomial differential system* or simply a *polynomial system* will be a differential system of the form

$$\frac{dx}{dt} = \dot{x} = p(x, y), \qquad \frac{dy}{dt} = \dot{y} = q(x, y), \tag{1}$$

where the dependent variables x and y are complex, the independent one (the *time*) t is real, and p and q are polynomials in the variables x and y with complex coefficients. In all this paper $m = \max\{\deg p, \deg q\}$ will denote the *degree* of the polynomial system.

[1] The authors are partially suported by a DGICYT Grant PB96–1153 and by CICYT Grant 1999SGR 00349.

0022-0396/01 $35.00