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Algebraic limit cycles of degree 4 for quadratic systems

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Abstract

Yablonskii (Differential Equations 2 (1996) 335) and Filipstov (Differential Equations 9 (1973) 983) proved the existence of two different families of algebraic limit cycles of degree 4 in the class of quadratic systems. It was an open problem to know if these two algebraic limit cycles where all the algebraic limit cycles of degree 4 for quadratic systems. Chavarriga (A new example of a quartic algebraic limit cycle for quadratic systems, Universitat de Lleida, Preprint 1999) found a third family of this kind of algebraic limit cycles. Here, we prove that quadratic systems have exactly four different families of algebraic limit cycles. The proof provides new tools based on the index theory for algebraic solutions of polynomial vector fields.

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Keywords: Algebraic limit cycles; Quadratic systems; Quadratic vector fields

1. Introduction

We consider polynomial differential systems

$$\dot{x} = P(x, y), \quad \dot{y} = Q(x, y), \quad (1)$$

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