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Weakened Markus–Yamabe conditions for 2-dimensional global asymptotic stability

Marc Chamberland[a], Jaume Llibre[b,*], Grzegorz Świrszcz[c]

[a] *Department of Mathematics and ComputerScience, Grinnell College, Grinnell, IA, 50112, USA*
[b] *Departament de Matemàtiques, Facultat de Ciencies, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Spain*
[c] *Institute of Mathematics, University of Warsaw, Banacha 2,02-097 Warsaw, Poland*

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Abstract

For a general 2-dimensional autonomous system $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$, it is difficult to find easily verifiable sufficient conditions guaranteeing global asymptotic stability of an equilibrium point. This paper considers three conditions which imply global asymptotic stability for a large class of systems, weakening the so-called Markus–Yamabe condition. The new conditions are: (1) the system admits a unique equilibrium point, (2) it is locally asymptotically stable, and (3) the trace of the Jacobian matrix of $\mathbf{f}$ is negative everywhere. We prove that under these three conditions global asymptotic stability is obtained when the components of $\mathbf{f}$ are polynomials of degree two or represent a Liénard system. However, we provide examples that global asymptotic stability is not obtained under these conditions for other classes of planar differential systems.

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1. Introduction and statement of the main results

Since the time of Liapunov, it has become evident that finding conditions which guarantee global asymptotic stability of an equilibrium point in a differential system, even in

* Corresponding author. Tel.: +31-93-581-13-03; fax: +34-93-581-27-90.
E-mail addresses: chamberl@math.grin.edu (M. Chamberland), jllibre@mat.uab.es (J. Llibre), swirszcz@mimuw.edu.pl (G. Świrszcz).

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