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Quadratic systems with limit cycles of normal size

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Abstract. In the class of planar autonomous quadratic polynomial differential systems we provide 6 different phase portraits having exactly 3 limit cycles surrounding a focus, 5 of them have a unique focus. We also provide 2 different phase portraits having exactly 3 limit cycles surrounding one focus and 1 limit cycle surrounding another focus. The existence of the exact given number of limit cycles is proved using the Dulac function. All limit cycles of the given systems can be detected through numerical methods; i.e. the limit cycles have "a normal size" using Perko's terminology.

Mathematics subject classification: 34C07, 34C08.
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1 Introduction

A planar autonomous quadratic polynomial differential system (or simply a *quadratic system*) in what follows is a system of the form

$$\frac{dx}{dt} = \sum_{i+j=0}^{2} a_{ij}x^i y^j \equiv P(x,y), \quad \frac{dy}{dt} = \sum_{i+j=0}^{2} b_{ij}x^i y^j \equiv Q(x,y), \tag{1}$$

with $a_{ij}, b_{ij} \in \mathbb{R}$. It is known (see, for instance [17]) that a quadratic system can have only limit cycles enclosing a unique singular point, which is a focus. As system (1) has no more than two foci [17], only the following distributions of limit cycles are allowed: n, (n_1, n_2), where $n \in \mathbb{N}$, and $n_1, n_2 \in \mathbb{N} \cup \{0\}$ with $n_1 + n_2 > 0$. Here n is the number of limit cycles surrounding a focus provided that system (1) has only one focus, and n_1 and n_2 are the number of limit cycles surrounding every one of the two foci provided that the system has exactly two foci. Recently, Zhang Pingguang [20, 21] has proved that if $n_i > 0$ for $i = 1, 2$, then either $n_1 = 1$, or $n_2 = 1$.

The following distributions of limit cycles for quadratic systems (1) are known:

(a) 1 and $(1,0)$; (b) 2 and $(2,0)$; (c) 3 and $(3,0)$;
(d) $(1,1)$; (e) $(2,1)$; (f) $(3,1)$.

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