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# THE CYCLICITY OF PERIOD ANNULI OF SOME CLASSES OF REVERSIBLE QUADRATIC SYSTEMS

G. Chen[1], C. Li[2], C. Liu[1,2] and J. Llibre[3]

[1]UFR de Mathématiques, Laboratoire P. Painlevé, UMR 8524
Université de Lille 1, 59655 Villeneuve d'Ascq, France

[2]LMAM and School of Mathematical Science, Peking University
Beijing 100871, P.R.China

[3]Departament de Matemàtiques, Universitat Autònoma de Barcelona
Bellaterra, 08193 Barcelona Spain

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Abstract. The cyclicity of period annuli of some classes of reversible and non-Hamiltonian quadratic systems under quadratic perturbations are studied. The argument principle method and the centroid curve method are combined to prove that the related Abelian integral has at most two zeros.

1. **Introduction.** As a part of the study of Hilbert's 16th problem, many authors considered the quadratic perturbations of quadratic centers. If the quadratic centers belong to the Hamiltonian class, then the study of the number of limit cycles bifurcating from a period annulus or annuli (*i.e.* the weak Hilbert's 16th problem for $n = 2$) is finished, and the study of the number of limit cycles bifurcating from singular loop, or from infinity is partially finished, see [7, 10, 27, 6, 17, 2, 11, 12, 8, 28, 29, 3, 15, 9, 16]. If the quadratic centers belong to the reversible class (and do not belong to the Hamiltonian class), then the study seems very difficult, and known results are very limited (see a list after Remark 1.1, and see [24] for more information about the weak Hilbert's 16th problem).

In the present paper we shall study perturbations of some classes of generic quadratic reversible and non-Hamiltonian systems. In [22, 30] a classification is given for integrable quadratic systems with at least one center. Following [13], such systems can be classified into five classes in the complex form:

| | | |
|---|---|---|
| $(i)$ | $\dot z = -iz - z^2 + 2\lvert z\rvert^2 + (\bar b + i\bar c)\bar z^2,$ | Hamiltonian $(Q_3^H)$ |
| $(ii)$ | $\dot z = -iz + \bar a z^2 + 2\lvert z\rvert^2 + \bar b \bar z^2,$ | reversible $(Q_3^R)$ |
| $(iii)$ | $\dot z = -iz + 4z^2 + 2\lvert z\rvert^2 + (\bar b + i\bar c)\bar z^2,$ | $\lvert\bar b + i\bar c\rvert = 2,$ codimension four $(Q_4)$ |
| $(iv)$ | $\dot z = -iz + z^2 + (\bar b + i\bar c)\bar z^2,$ | generalized Lotka – Volterra $(Q_3^{LV})$ |
| $(v)$ | $\dot z = -iz + \bar z^2.$ | Hamiltonian triangle |

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