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J. Differential Equations 269 (2020) 832–861

**Journal of  
Differential  
Equations**

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# $Z_2$ -equivariant linear type bi-center cubic polynomial Hamiltonian vector fields

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Received 12 August 2019; revised 1 December 2019; accepted 26 December 2019  
Available online 10 January 2020

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## Abstract

We study the global dynamical behavior of  $Z_2$ -equivariant cubic Hamiltonian vector fields with a linear type bi-center at  $(\pm 1, 0)$ . By using a series of symbolic computation tools, we obtain all possible phase portraits of these  $Z_2$ -equivariant Hamiltonian systems.

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*MSC:* primary 34C07, 34C08

*Keywords:*  $Z_2$ -equivariant; Hamiltonian; Phase portrait; Bi-center; Infinity

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## 1. Introduction and statement of the main results

In the qualitative theory of planar vector fields the analysis when an equilibrium point  $p$  is either a center or a focus is one of the classical problems. We called it the center problem or the center-focus problem. Poincaré [27] and Dulac [13] defined that an equilibrium point  $p$  of a vector field in  $\mathbb{R}^2$  is a *center* if it has a neighborhood  $U$  filled with periodic orbits with the unique exception of this equilibrium point. And a center  $p$  is *global* if  $\mathbb{R}^2 \setminus \{p\}$  is filled with periodic orbits.

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