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# A unified proof on the weak Hilbert 16th problem for $n = 2^\star$

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## Abstract

The weak Hilbert 16th problem for  $n = 2$  was solved by Horozov and Iliev (Proc. London Math. Soc. 69 (1994) 198–244), Zhang and Li (Adv. in Math. 26 (1997) 445–460), (Gavrilov Invent. Math. 143 (2001) 449–497), and Li and Zhang (Nonlinearity 15 (2002) 1775–1992), by using different methods for different cases. The aim of this paper is to give a unified and easier proof for all cases. The proof is restricted to the real domain, combines geometric and analytical methods, and uses deformation arguments.

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