BIFURCATION DIAGRAMS AND GLOBAL PHASE PORTRAITS FOR SOME HAMILTONIAN SYSTEMS WITH RATIONAL POTENTIALS

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ABSTRACT. In this paper we study the global dynamical behavior of the Hamiltonian system $\dot{x} = H_y(x, y)$, $\dot{y} = -H_x(x, y)$ with the rational potential Hamiltonian $H(x, y) = y^2/2 + P(x)/Q(y)$, where $P(x)$ and $Q(y)$ are polynomials of degree 1 or 2. First we get the normal forms for these rational Hamiltonian systems by some linear change of variables. Then we classify all the global phase portraits of these systems in the Poincaré disk and provide their bifurcation diagrams.

1. Introduction and statement of the main results

A great deal of work has been done for studying the global dynamics of the planar polynomial differential systems, for example see [3, 4, 6, 8, 10, 15]. Vulpe [16] studied the global phase portraits of the quadratic polynomial systems having a center. In [14] Schlomiuk gave the bifurcation diagrams for the global phase portraits of these quadratic systems. Artés and Llibre [2] provided the global phase portraits of all quadratic Hamiltonian systems. The authors of [12] presented the phase portraits of the quadratic polynomial vector fields having a rational first integral of degree 3. Guillamon et al. [11] gave an algorithm to obtain the phase portraits of the separable Hamiltonian system with the Hamiltonian function $H(x, y) = F(x) + G(y)$. Colak et al. [5, 7] presented the global phase portraits in the Poincaré disk of all Hamiltonian linear type centers of polynomial systems having linear plus cubic homogeneous terms, and gave their bifurcation diagrams.

In this paper we consider the Hamiltonian system

$$\dot{x} = H_y(x, y), \qquad \dot{y} = H_x(x, y) \tag{1}$$

with a rational Hamiltonian function

$$H = H(x, y) = \frac{y^2}{2} + V(x, y) = \frac{y^2}{2} + \frac{P(x)}{Q(y)}, \tag{2}$$

where $P(x)$ and $Q(y)$ are real polynomials of degree at most 2. We denote by the set $L = \{(x, y)|Q(y) = 0\}$ the points where the Hamiltonian vector field are not defined. The system associated to the Hamiltonian function (2) has the form

$$\dot{x} = y - \frac{P(x)Q'(y)}{Q^2(y)}, \qquad \dot{y} = -\frac{P'(x)}{Q(y)}, \tag{3}$$

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