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Global dynamics of a SD oscillator

Hebai Chen · Jaume Llibre · Yilei Tang

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Abstract In this paper, we derive the global bifurcation diagrams of a SD oscillator which exhibits both smooth and discontinuous dynamics depending on the value of a parameter a. We research all possible bifurcations of this system, including Pitchfork bifurcation, degenerate Hopf bifurcation, homoclinic bifurcation, double limit cycle bifurcation, Bautin bifurcation and Bogdanov–Takens bifurcation. Besides, we show that the system has five limit cycles, including four small limit cycles and one large limit cycle. At last, we give all numerical phase portraits to illustrate our results.

Keywords SD oscillator · Homoclinic loop · Limit cycle · Hopf bifurcation · Bogdanov–Takens bifurcation · Averaging method

Mathematics Subject Classification 34C29 · 34C25 · 47H11

H. Chen
College of Mathematics and Computer Science, Fuzhou University, Fuzhou 350002, Fujian, People's Republic of China
e-mail: chen_hebai@sina.com

J. Llibre (✉)
Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona, Catalonia, Spain
e-mail: jllibre@mat.uab.cat

Y. Tang
School of Mathematical Sciences, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
e-mail: mathtyl@sjtu.edu.cn

1 Introduction and main results

In recent years, SD (smooth and discontinuous, for short) oscillator was proposed and investigated for studying the transition from smooth to discontinuous dynamics, see, for instance [2–5,15]. In [5,15], the van der Pol damped SD oscillator is given by

$$\ddot{x} + \xi(b + x^2)\dot{x} + x\left(1 - \frac{1}{\sqrt{x^2 + a^2}}\right) = 0 \tag{1}$$

for studying this transition, where $a \geq 0$, b and ξ can take arbitrary real values. More precisely, the smooth dynamics appears when $a > 0$, while the discontinuous dynamic behavior occurs at $a = 0$. The global dynamics was completely studied in [5] when $a = 0$, and in [15] when $|a - 1| < \varepsilon$, $|\xi| < \varepsilon$ and ε are sufficiently small. Besides, Cao et al. in [2–4] studied the following linear damped SD oscillator with periodic excitation

$$\ddot{x} + 2\xi\dot{x} + x\left(1 - \frac{1}{\sqrt{x^2 + a^2}}\right) = f_0\cos(\omega t) \tag{2}$$

using the fact that when $f_0 = 0$ this system admits a Hamiltonian formulation, and they analyze the dynamics of the perturbed Hamiltonian system by Melnikov methods when f_0 is small. Moreover, they gave many numerical results. Clearly, the dynamics of periodic system (2) is different from the dynamics here studied, the one of system (1).

Clearly, system (1) can be rewritten as the 2-dimensional differential system