AVERAGING APPROACH TO CYCLICITY OF HOPF BIFURCATION IN PLANAR LINEAR-QUADRATIC POLYNOMIAL DISCONTINUOUS DIFFERENTIAL SYSTEMS

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ABSTRACT. It is well known that the cyclicity of a Hopf bifurcation in continuous quadratic polynomial differential systems in $\mathbb{R}^2$ is 3. In contrast here we consider discontinuous differential systems in $\mathbb{R}^2$ defined in two half–planes separated by a straight line. In one half plane we have a general linear center at the origin of $\mathbb{R}^2$, and in the other a general quadratic polynomial differential system having a focus or a center at the origin of $\mathbb{R}^2$. Using averaging theory, we prove that the cyclicity of a Hopf bifurcation for such discontinuous differential systems is at least 5. Our computations show that only one of the averaged functions of fifth order can produce 5 limit cycles and there are no more limit cycles up to sixth order averaged function.

1. Introduction and statement of the main result

A *polynomial differential system* in $\mathbb{R}^2$ is a differential system of the form

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

where P and Q are polynomials in the real variables x and y, and the dot denotes derivative with respect to an independent variable t. The *degree* of the polynomial differential system (1) is the maximum of the degrees of the polynomials P and Q.

A *linear differential system* or here simply a *linear system* is a polynomial differential system in $\mathbb{R}^2$ of degree one. Similarly a *quadratic differential system* or simply a *quadratic system* is a polynomial differential system in $\mathbb{R}^2$ of degree two.

A *limit cycle* of a differential system is a periodic orbit of that system which is isolated in the set of all periodic orbits of the system. The study of the limit cycles of the planar differential systems is one of the main topics of the qualitative theory of the differential systems in $\mathbb{R}^2$, see for instance [9, 18, 22].

A *Hopf bifurcation* takes place at a singular point p of a differential system when p changes its stability and one or several limit cycles arise from p. Here the *cyclicity of a Hopf bifurcation* at the singular point p inside a family of differential systems is the maximum number of limit cycles which can bifurcate from p inside the family considered.

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