Algebraic Aspects of Integrability for Polynomial Systems

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We present an introductory survey to the Darboux integrability theory of planar complex and real polynomial differential systems. Our presentation contains some improvements to the classical theory.

Key Words: Darboux integrability, exponential factors, invariant algebraic curves.

1. INTRODUCTION

The main part of these notes is devoted to explaining the fascinating connection between the local integrability of polynomial differential equations (a topological phenomena) and the existence of exact solutions for these equations (an algebraic one). However, having built up the appropriate methods, it seemed a good idea to digress into a few closely related areas.

There are now several expositions of the Darboux method of integration [9, 20, 1, 6], so our aim here is not at completeness, but at obtaining a general idea of the methods involved. Once the geometric ideas are grasped (and these are not difficult) the more technical theorems are mostly routine. Some of these have been left as exercises in the text.