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INTEGRABILITY VIA INVARIANT ALGEBRAIC CURVES FOR PLANAR POLYNOMIAL DIFFERENTIAL SYSTEMS *

Colin Christopher
(School of Mathematics and Statistics, University of Plymouth, Plymouth, Devon PL4 8AA, UNITED KINGDOM)
Jaume Llibre
(Departament de Matemàtiques, Universitat Autònoma de Barcelona, 08193 - Bellaterra, Barcelona, SPAIN)

Abstract

We present an introduction to the Darboux integrability theory of planar complex and real polynomial differential systems containing some improvements to the classical theory.

Keywords integrability, invariant algebraic curves, planar polynomial differential systems

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1 Introduction

By definition a *complex* (respectively *real*) *planar polynomial differential system* or simply a *polynomial system* is a differential system of the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \dot{x} = P(x, y), \qquad \frac{\mathrm{d}y}{\mathrm{d}t} = \dot{y} = Q(x, y), \tag{1}$$

where the dependent variables x and y are complex (respectively real), the independent one (the *time*) t is real, and P and Q are polynomials in the variables x and y with complex (respectively real) coefficients. In all this paper $m = \max\{\deg P, \deg Q\}$ denote the *degree* of the polynomial system.

In this work we want to show the fascinating relationships between the integrability (a topological phenomena) and the existence of exact algebraic solutions for polynomial systems. In 1878 Darboux [9] showed how first integrals can be constructed for polynomial systems possessing sufficient invariant algebraic curves (see Section 6). In particular, he proved that if a polynomial

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