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Inverse problems for invariant algebraic curves: explicit computations

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Given an algebraic curve in the complex affine plane, we describe how to determine all planar polynomial vector fields which leave this curve invariant. If all (finite) singular points of the curve are non-degenerate, we give an explicit expression for these vector fields. In the general setting we provide an algorithmic approach, and as an alternative we discuss sigma processes.

1. Introduction

The question of determining the invariant algebraic curves of a given planar polynomial vector field, or to decide that no such curves exist, is part of a problem set forward by Poincaré, and is also essential in deciding whether the vector field admits a Darboux integrating factor. While there are many partial answers to this question, a complete solution to the problem still seems beyond reach.

A solution of the inverse problem, i.e. to determine all polynomial vector fields that admit a prescribed collection of invariant algebraic curves (or just one possibly reducible curve), seems to be essential in order to obtain a proper understanding of the situation. We present a solution to this inverse problem, using mainly results and tools from elementary commutative algebra. Our results indicate that it is sensible to consider the affine plane in its own right, as well as as a subset of the projective plane. For the sake of completeness and to emphasize that our approach works very naturally, we also include streamlined alternative proofs for some theorems that have been published previously.